The Utility of Multi-Objective Conditioning of a Distributed Hydrological Model Using Uncertain Estimates of Saturated Areas

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Abstract

The inability to reliably measure the distributed physical characteristics of a catchment results in significant uncertainty in the parameterisation of physically based, distributed models. Calibration of such models is usually achieved with limited discharge data. Due to the parameteric complexity of such models, robust calibration of parameters is not achieved as these models are over-parameterised; many combinations of parameter values from many areas of the parameter space may produce simulations that fit the observed flow data reasonably well. The parameteric uncertainty produces significant predictive uncertainty, in terms of the range of discharge predictions, but also in terms of internal states of acceptable model simulations; the interaction of processes through ill-defined parameter estimates permits the reproduction of the discharge hydrograph via a range of modelled process mechanisms. In this study, uncertain estimates of the internal behaviour of catchment response are used as an additional criterion in the assessment of the acceptability of model simulations. In two applications, it is shown that such uncertain estimates may greatly reduce the parametric uncertainty associated with acceptable parameterisations, and hence the predictive uncertainty of such models. This is achieved through the improved definition of internal processes afforded by the uncertain estimates. Additionally, as it is shown that multi-objective criteria may be used to constrain parameter estimates by model inversion, investigation of multi-scale hydraulic permeabilities is permitted. The scaling behaviour observed in one such investigation indicates that preferential flow pathways dominate the characteristic permeability of the catchment response.

1. INTRODUCTION

In an attempt to represent the physics of the hydrological behaviour of catchments, many complex distributed models exist, each requiring various levels of complexity in their parameterisation. Due to the inability to accurately measure distributed physical properties of environmental systems, calibration against observed data is typically performed. This is most often achieved with limited rainfall – runoff data. As noted by Beven (1989), given the complexity of such models, many different combinations of parameter values may simulate the discharge hydrograph equally well. Many parameterisations may therefore be deemed acceptable given the limited calibration data. These parameter sets may be located throughout many areas of the parameter space (Duan et al., 1992; Beven, 1993). This uncertainty of the appropriate parameter values yields predictive uncertainty as has been demonstrated through applications of the Generalised Likelihood Uncertainty Estimation (GLUE) methodology (Beven and Binley, 1992; Freer et al., 1996).

To constrain the acceptable parameter sets, additional discharge data sets may be used, hopefully incorporating more of the dynamics of the system. Alternatively, competing model parameterisations may be rejected on the basis of predictions of hydrological variables other than discharge. Saturated area predictions, in particular, reflecting the integrated response of hillslope processes, may be a useful discriminatory variable. If a given model parameterisation predicts saturation excess overland flow as a mechanism in the field, but it is known that this did not occur in reality, then the parameterisation may be rejected as the physical processes in the field are not being accurately reproduced in the model (Beven, 1989). Qualitative information of this nature may therefore be used to constrain model uncertainties. Likewise, and to a greater extent, quantitative information of saturated areas may be employed to discriminate between models, but quantitative data on saturated contributing areas, either surface or subsurface, is difficult to obtain by ground surveys (see for example, Dunne et al., 1975; Beven, 1978).

In this study, a physically based distributed model (TOPMODEL; Beven et al., 1995) is applied to two catchments. The first is the Naizin catchment in Northern France, the other is the Baru catchment, located in tropical Borneo. Rainfall – runoff data are employed in each application to assess the acceptability of multiple parameterisations drawn from a priori acceptable ranges, through a goodness of fit criterion. In the case of Naizin catchment, uncertain estimates of saturated areas are derived from ERS-1 SAR imagery (Gineste et al., 1997), and are then used as a second modelling objective thus providing a secondary discriminant of acceptable – unacceptable model parameterisations.

A similar approach is adopted for the application of TOPMODEL to the Baru catchment, though without the aid of remotely sensed data. Given an appreciation of the behaviour of the catchment from field experience and runoff plots, uncertain estimates of saturated areas for a given storm are utilised to constrain the acceptable parameterisations. The effect of the additional conditioning on the acceptable parameter response surfaces is then investigated for both applications.

2. TOPMODEL

TOPMODEL (Beven and Kirkby, 1979; Beven et al., 1995) has been developed with the aim of providing a simple but physically reasonable, distributed description of the processes involved in rainfall-runoff production requiring a minimum of parameters. Its primary premise is that the distributed responses of a catchment can be described through the use of the $\ln(a/\tan\beta)$ topographic index of hydrological similarity calculated from the upslope area draining through a point per unit contour length, a, and the slope at that point, $\tan\beta$. The popularity of TOPMODEL in recent years has lead to a wide range of applications (see for example, the review of Beven et al. [1995]).

TOPMODEL is not intended as a fixed modelling package, but rather a collection of concepts that can be used, modified or rejected as an application demands. One key development in the recent history of TOPMODEL is the generalisation of the storage - discharge relationship. Most TOPMODEL applications assumed an exponential relationship, whereas in a number of recent applications the model could be better fit to observed discharges though the use of parabolic, linear, or a compound function (Ambroise et al., 1996).

In the application to the Naizin catchment, preliminary modelling of the rainfall-runoff data indicated that a compound storage - discharge relationship should be adopted due to the particular baseflow recession of the Naizin catchment. Recessions were analysed using the Master Recession Curve analysis software (MRCtool) developed by Lamb [1996]. From this analysis, it was evident that the recession contained two components, both of which follow an exponential form of the storage - discharge relationship, but with distinctly different gradients - the recession of the flow at a discharge of 4×10^{-5} mh^{-1} abruptly and consistently changes. Following this analysis, a compound storage - discharge relationship was adopted consisting of two exponential functions above and below this threshold discharge.

3. THE GLUE METHODOLOGY

The Generalised Likelihood Uncertainty Estimation (GLUE) methodology (Beven and Binley, 1992) was developed as a method for calibration and uncertainty estimation of models based on generalised likelihood measures, and is an extension of the Generalised Sensitivity Analysis of Spear and Hornberger [1980]. The GLUE methodology attempts to explicitly recognise the fundamental limitations of representing the rainfall-runoff process with contemporary hydrological models. Its application so far has predominantly been in rainfall-runoff modelling (Beven and Binley, 1992; Beven, 1993; Freer et al. 1996), but has also been applied to assess the uncertainty associated with predictions of land surface to atmosphere fluxes (Franks and Beven, 1997), geochemical modelling (Zak et al., 1997) and flood inundation estimation (Romanowicz et al., 1996).

GLUE is based upon Monte-Carlo simulation; a large number of model runs are made, each with random parameter values selected from probability distributions for each parameter. The acceptability of each run is assessed by comparing predicted to observed discharges through some chosen likelihood measure. Runs that achieve a likelihood below a certain threshold may then be rejected as 'non-behavioural'. The likelihoods of these non-behavioural parameterisations are set to zero and are thereby removed from the subsequent analysis.

Following the rejection of non-behavioural runs, the likelihood weights of the retained runs are rescaled so that their cumulative total is 1.0. At each time step, the predicted output from the retained runs are likelihood weighted and ranked to form a cumulative distribution of the output variable from which chosen quantiles can be selected to represent the model uncertainty.

While GLUE is based on a Bayesian conditioning approach, the likelihood measure is achieved through a goodness of fit criterion as a substitute for a more traditional likelihood function. This is due to the difficulties associated with environmental models where it is not generally possible to assume that any available model structure is correct, nor that a consistent error model can be defined (Franks and Beven, 1997). Nevertheless, the GLUE methodology retains the spirit of traditional likelihood theory, but with a more lenient likelihood function.

The likelihood value is associated with a particular set of parameter values within a given model structure. The likelihood associated with a particular parameter value may therefore be expected to vary depending on the values of the other parameters and there may be no clear optimum parameter set. Interaction between parameter values will be reflected implicitly in the likelihood values associated with the parameter sets. Multiple model structures may also compete to be considered acceptable within this framework. In most applications no direct account is taken of uncertainties in the input and boundary data driving the model; any such uncertainties are also implicitly reflected in the likelihood values as the likelihood values are conditioned on the inputs.

For the application of the GLUE methodology, the likelihood function adopted here to weight simulations according to their reproduction of the observed discharge time series is given by:

$$L(\underline{\Theta}_i \mid \underline{Y}) = (1 - \sigma_i^2 / \sigma_{obs}^2) \qquad (1)$$

where $L(\underline{\Theta}_i|\underline{Y})$ is the likelihood, σ_i^2 is the variance of the errors for parameter set $\underline{\Theta}_i$ given the set of discharge observations $\underline{Y}$, and σ^2_{obs} is the variance of the observed data set. This likelihood measure is equal to a coefficient of determination or the Nash and Sutcliffe [1970] efficiency.

3.1 Updating uncertainty estimates using Bayes equation

In order to constrain uncertainty, additional data may be incorporated into the likelihood estimates using the GLUE methodology. The updating of the likelihood distribution may be achieved through the application of Bayes Equation in the form:

$$L(\underline{\Theta}_i|\underline{Y}) = L_1(\underline{Y}|\underline{\Theta}_i) L_0(\underline{\Theta}_i) / C \qquad (2)$$

where $L_0(\underline{\Theta}_i)$ is a prior likelihood measure for the parameter set $\underline{\Theta}_i$; $L_1(\underline{Y}|\underline{\Theta}_i)$ is the likelihood measure calculated for the simulation of observed variable $\underline{Y}$ by the parameter set $\underline{\Theta}_i$; $L(\underline{\Theta}_i|\underline{Y})$ is the posterior likelihood for the parameter set $\underline{\Theta}_I$ given the new observations $\underline{Y}$; and C is a scaling constant. Application of (2) in this form ignores any correlation of the simulations of the different parameter sets. The parameter sets are chosen independently, but the simulated variables (and resulting likelihood measures) may be correlated due to forcing of the model dynamics by the input variables.

In application to both catchments, model parameter sets are evaluated with respect to the comparison of predicted and observed discharges through application of equation (1).

For the application to the Naizin catchment, Bayesian updating of likelihoods is then performed through application of equation (2) with respect to the predicted areal extent of saturation compared to the uncertain range of actual saturated area extent as inferred from the extrapolation of the limited saturation ground truth through the joint use of the topographic index and an index of saturation potential gained from multiple SAR images (Gineste et al., 1997). The updated, likelihood weighted simulations may then be used to derive the uncertainty bounds. In comparison to the uncertainty bounds derived through conditioning on discharges alone, an appreciation of the utility of the additional constraint may be gained.

4. APPLICATION OF GLUE TO THE NAIZIN CATCHMENT

Feasible parameter ranges were selected for the Naizin catchment and 10000 parameter sets were selected. TOPMODEL was then run for all parameter sets, the model efficiency produced being associated with that parameter set. Figure 1 shows the scattergrams for the application of TOPMODEL to the Naizin catchment where the likelihood associated with each model parameterisation is achieved through conditioning on the observed discharge time series alone.

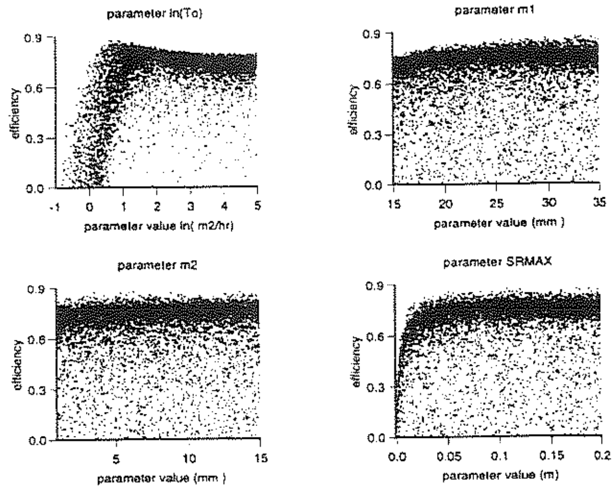

Figure 1. Scattergrams for four TOPMODEL parameters plotted against model efficiency.

Limited ground truth data were available for a subcatchment of the Naizin basin providing some estimate of the extent of saturated areas. Utilising an approach based upon the combination of the topographic index and the Saturation Potential Index (Gineste et al., 1997) the ground truth data were extrapolated to yield estimates of the total catchment saturated area extent (Franks et al., 1997). As this extrapolation is inherently uncertain, multiple combinations of the topographic index – SPI were used and a range of feasible saturated areas produced. Figure 2 shows a map of the distribution of saturated areas as predicted by one combination of the topographic and saturation potential indices. Figure 3 shows the derived range of saturated areas for the Naizin catchment for a given storm incorporating the uncertainty associated with the extrapolation from the limited ground truth to the catchment wide area. The derived likelihood weighted saturated areas may therefore be used as a secondary modelling objective against which model simulations can be compared or rejected.

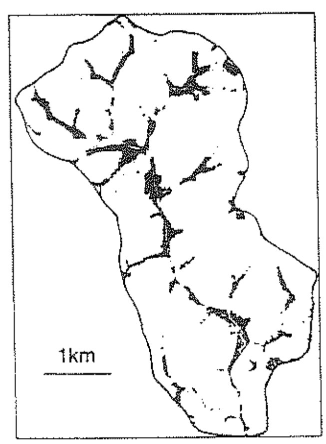

Figure 2. Derived prediction of saturated area from the extrapolation of combined topographic index and saturation potential index, thresholded against ground truth data

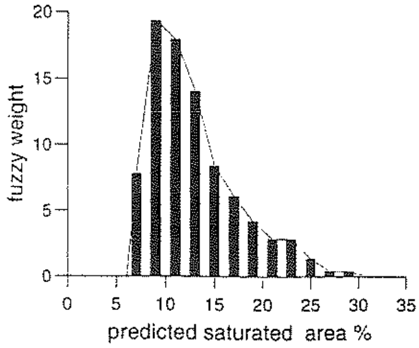

Figure 3. Derived estimates of catchment wide saturated areal extent

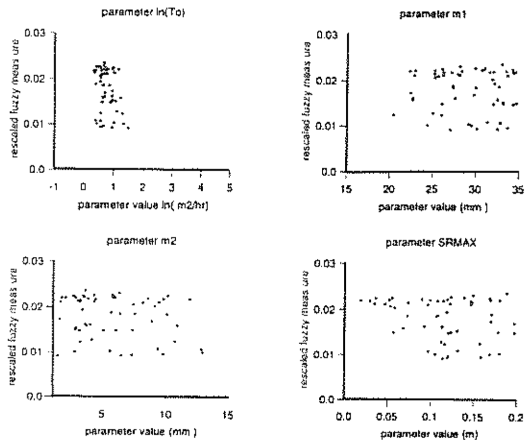

Figure 4. Scattergrams after conditioning parameter sets using the additional saturation criteria

Figure 4 shows the same scattergrams after the additional conditioning afforded by the derived uncertain estimates of the saturated areas. As can be seen by comparison with Figure 1, the imposition of additional modelling objectives has yielded significant modification of the acceptable parameter sets. Most notably, the catchment effective saturated transmissivity parameter has been shown to be greatly constrained through such conditioning.

4.1 Predictive uncertainty

Through application of the GLUE methodology, the predictive uncertainty associated with the multiple acceptable parameter sets may be derived. The utility of the saturated areal extent as an additional modelling objective is graphically demonstrated in Figure 5. Following the GLUE methodology, uncertainty bounds are shown when propagated according to the model simulations deemed acceptable according to the reproduction of the discharge time series alone (i.e. through the model efficiency alone), and then updated with the additional saturated area constraint.

As can be seen, a marked difference in the derived uncertainty bounds is achieved. The effect of the additional constraint is to (i) narrow the range of the uncertainty bounds, therefore indicating reduced predictive uncertainty, and (ii) produce improved uncertainty estimates as indicated by the bounds encompassing more of the discharge hydrograph (though note that in the case of Figure 5b the observed record has been omitted due to apparent error in the data).

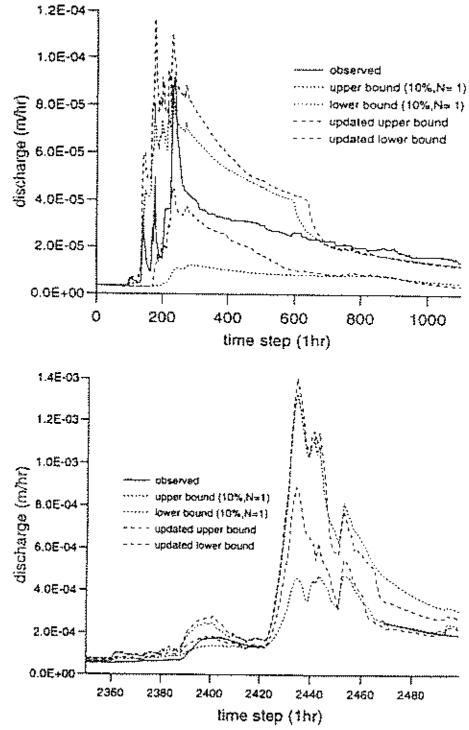

Figure 5 a,b. Predictive uncertainty bounds after conditioning on reproduction of the discharge time series alone, and reproduction of the discharge time series and updated with the range of likely catchment wide saturated areal extent.

5 APPLICATION OF UNCERTAIN SATURATION CONSTRAINT TO THE BARU CATCHMENT

Feasible parameter ranges were specified for the Baru catchment from which 10000 model parameter sets were uniformly sampled. TOPMODEL was run with these parameter sets, and a model efficiency was associated with each set according to the reproduction of the discharge hydrograph. Figure 6 shows the scattergrams for four of the TOPMODEL parameters after conditioning on discharges alone. As can be seen, the scattergrams show that good fits to the discharge data may be achieved with very diverse values of each of the parameters.

Figure 7 shows the modified scattergrams of acceptable parameter sets after the rejection of parameter set which produced saturated areas not deemed realistic (i.e. outside the range of 2 and 10%). As can be seen, significant constraint of the transmissivity parameter occurs as a function of this additional, albeit uncertain, conditioning. It should be noted that the global optimum parameter set in terms of the efficiency has been rejected due to its unreasonably high prediction of saturated contributing area.

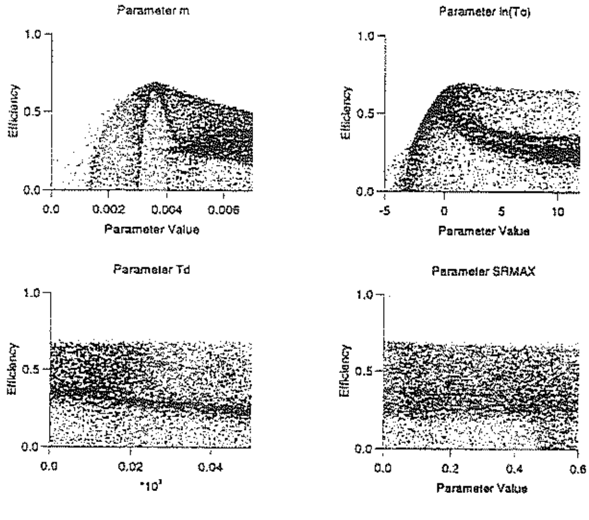

Figure 6. Scattergrams for four of the TOPMODEL parameters plotted against model efficiency.

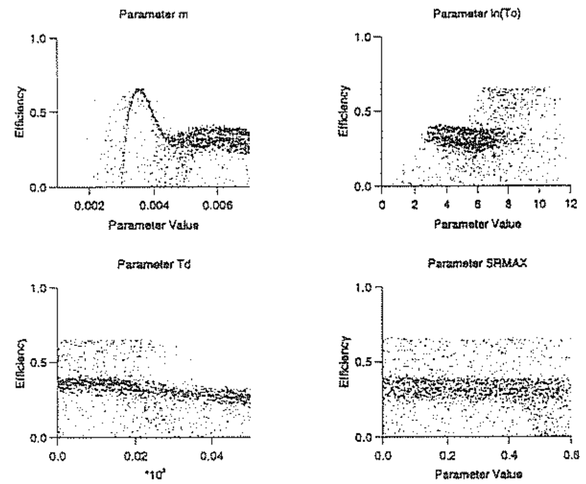

Figure 7. Scattergrams for the four TOPMODEL parameters after additional evaluation of each of the simulations with respect to the predicted saturated areal extent. Note the marked constraint of both the m and ln(T0) parameters.

5.1 Comparison of multi-scale permeabilities

By applying uncertain saturation criterion to the conditioning process, it has been shown that constrained ranges of the transmissivity parameter have resulted. At the Baru site, extensive core-scale permeability measurements have been undertaken with a ring based permeameter (Chappell and Ternan, 1997), in addition to hillslope-scale permeabilities derived by tracer tests. Thus, the constrained model inversion enables the comparison of the range of model derived permeability estimates over the depth of the saturated media after correction for the effects of media depth and effective porosity (Chappell et al., 1997).

Choosing the maximum and minimum values of the catchment effective saturated transmissivity and corresponding values of the m parameter, from those simulations deemed acceptable with an efficiency greater than 0.5, the range of model inverted saturated permeability was found to be 0.527×10^{-6} to 13.7×10^{-6} m/s. Core based measurements displayed a smaller range between 0.158×10^{-6} to 0.311×10^{-6} m/s – clearly less than the model derived range, whilst 'pulse-wave' experiments on an hillslope of the catchment yielded a range of permeabilities from 8.2×10^{-6} to 13.6×10^{-6} m/s, which are more consistent with the model results.

These results indicate differences between measures of permeability at a range of scales. Both hillslope and model inverted estimates are considerably higher than the core based measures, indicating that there may be a greater non-linearity of the catchment response than can be inferred from core based measurements alone. Indeed, the higher estimates of both the model and the hillslope measures are consistent with the occurrence and dominance of soil pipe processes. Though any firm conclusion is tentative, the additional constraint of feasible model parameterisations may provide increased confidence in the inversion of models within an uncertainty framework for such comparisons and may provide insight into catchment process behaviours.

6. DISCUSSION

The complexity of the processes to be represented for environmental management requires parameteric complexity of process orientated models. Given the inability to measure the distributed catchment characteristics accurately, calibration is therefore required. For the purposes of environmental management, if a process based model is required, then it is also required that those processes are adequately defined. As has been shown, discharge time series do not provide adequate information for the robust calibration of process hydrological models – one can reproduce the discharge time series reasonably well for the calibration period from many areas of the parameter space, and hence with a range of process representations. For instance, it is has been shown that a global optimum parameter set (in terms of the reproduction of the discharge) might be rejected through consideration of the internal states, such as the contributing saturated area. This parameteric uncertainty leads to a wide range of predictive capability.

The application of the GLUE methodology to the Naizin catchment has shown that significant uncertainty must be associated with the predictions of such models when calibrated against discharge data alone. Through the specification of a secondary modelling objective, the parameteric uncertainty has been greatly reduced, most markedly in the constraint of the catchment effective saturated transmissivity parameter. Additionally, this reduction in parameteric uncertainty translates into a marked reduction in the predictive uncertainty: when uncertainty bounds were propagated for the constrained parameter sets, a marked and consistent reduction of predictive uncertainty resulted from the additional constraint. Similarly, the imposition of an additional modelling objective on the conditioning of TOPMODEL for the Baru catchment has been shown to lead to markedly constrained parameter ranges of both the catchment effective saturated transmissivity parameter, and also the gradient of the exponential storage-discharge relationship (parameter m).

The utility of uncertain estimates of saturated area on the conditioning of physically based models has therefore been shown to be significant. Despite uncertainty in the specification of the additional constraint, more robust model conditioning may be achieved within an uncertainty framework.

The results of this work indicate that continuous measurement of discharge, whilst useful, is not sufficient to identify suitable model parameterisations. Additional information is required. This work also indicates that for application to an unguaged catchment, multiple parameterisations might be better identified given information pertaining to the specification of the dominant flow producing mechanisms, such as an uncertain appreciation of the hydrograph recession and the uncertain quantification of the maximum discharge and saturated area extent for a given storm or storm period.

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